Primary Mathematics Challenge – November 2021

Answers and Notes

These notes provide a brief look at how the problems can be solved.
There are sometimes many ways of approaching problems, and not all can be given here.
Suggestions for further work based on some of these problems are also provided.

P1 **E** $(2 + 0 + 2 + 1 = 5)$ P2 **C**  (an ellipse)

1 **B** 10p — Ten dinosaurs at 99p each will cost £9.90, so there will be 10p change.

2 **C** 4 m — Unless your car is a very stretched limousine, its length will be nearest to 4 m.

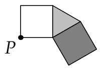
3 **C** 3 m — Twice 150 cm is 300 cm, so Nellie is about 3 m tall.

4 **B** Tuesday — Emma will take a week and 4 days; working backwards from Friday is a Tuesday.

5 **D** 7 — We can see that $\frac{1}{4}$ replied "no" and $\frac{3}{4}$ "yes". Since 21 replied "yes", the number that replied "no" was $21 \div 3 = 7$.

6 **C** 16:34 — The sun will be in the sky for 20 minutes less than 10 hours. Therefore it will set at 06:54 + 10 hours − 20 minutes = 16:34.

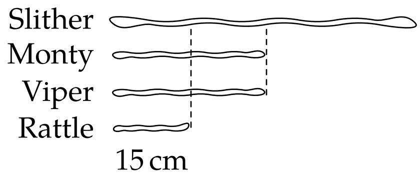
7 **D** ƧMIWƧ — The only letters *without* vertical symmetry are the two Ss, so the word will appear to be spelt backwards but with the Ss back-to-front, taking care to observe the differences between the W and the M. Only option D has all these details correct.

8 **B** 

After having been rotated by 90° clockwise, the triangle will 'point' to the right, which eliminates options A, C and D. The shaded square will also have been rotated, placing it below the triangle, hence option B.

9 **D** 24 — Two-thirds of the 36 sheep were dyed pink: $\frac{2}{3}$ of $36 = 24$.

10 **B** 30 cm — As this diagram shows, Monty and Viper have the same length, and Rattle, at 15 cm, is half as long. So Monty is 30 cm long.



11 **E** £6.30 — There are 28 days in four weeks, and each bag of salad will last four days. Therefore Jonathan will require seven bags over the four weeks. This will cost Jonathan or his owners 7×90p = £6.30.

12 **B** 12 — If we refer to the three children as Abel, Bron and Cora (**A**, **B** and **C**), then we can find six ways of arranging them in the middle of the sofa:

ABC **ACB** **BAC** **BCA** **CAB** **CBA**

For each of these 6 ways there are 2 ways for Pat (P) and Sam (S) to arrange themselves at either end of the sofa.

P**ABC**S P**ACB**S P**BAC**S P**BCA**S P**CAB**S P**CBA**S
S**ABC**P S**ACB**P S**BAC**P S**BCA**P S**CAB**P S**CBA**P

So there are $2 \times 6 = 12$ ways. Another approach is outlined in the Notes below.

13 **B** 3 hours — Mr Waddle takes $12 \div 2 = 6$ hours and the rather antisocial Mr Ramble takes $12 \div 4 = 3$ hours, so finishing 3 hours before Mr Waddle.

14 **E** multiple of 3 — Just trying a few examples $1 + 2 + 3$, $2 + 3 + 4$, etc will show that the totals are always a multiple of 3. This is because the least number and the greatest number, being 1 less and 1 greater respectively than the middle number will have a total of twice the middle. The total is therefore always three times that of the middle.

15 **C** 30 — At each lining up there are no children left over, so the number of children is a multiple of 2, 3 and 5. The smallest such number is $2 \times 3 \times 5 = 30$.

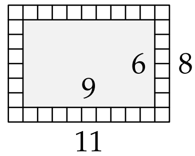
16 **A** 10 — This is a classic 'handshake' problem, and there are several methods of approaching it. One way is to take each girl in turn: Ava makes 4 high-fives with the other four girls; Bina has only 3 girls still to high-five; likewise Clare has only 2 remaining, and

after Dora has made a high-five with Elph, each girl has high-fived each of the others. Hence $4 + 3 + 2 + 1 = 10$ high-fives altogether. The other method is outlined in the Notes below.

17 **E** 39 When Isaac's mother is three times Isaac's age, the difference between their ages must be twice that of Isaac's age. The difference between their ages is $81 - 53 = 28$ and does not change. Therefore Isaac was 14, which was $53 - 14 = 39$ years ago.

18 **D** 54 There are several ways to approach this. One method is to count carefully the tiles that **do** touch the walls, taking care not to count the corner tiles twice (namely $2 \times 11 + 2 \times 8 - 4 = 34$) and then subtract that number from the total number of tiles on the floor. Hence the required number of tiles is $11 \times 8 - 34 = 54$.

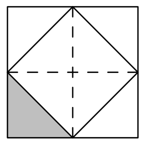
A quicker method is to notice that the tiles that do not touch the walls form a rectangle that has dimensions $(11 - 2) \times (8 - 2)$, so that there are $9 \times 6 = 54$ of them.



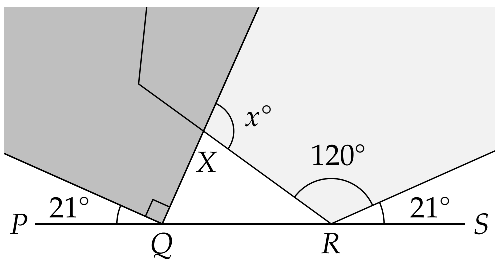
19 **A** 10p and 1p The total value of Penny's six coins is $(1 + 2 + 5 + 10 + 20 + 50)$p = 88p. But the separate amounts in the circle and in the square together amount to $(61 + 38)$p = 99p, which means that $(99 - 88)$p = 11p has been counted twice in the intersection. The only two coins that make up 11p are the 10p and the 1p.

20 **A** $\frac{1}{3}$ Each of the four shaded squares at the corners is $\frac{1}{16}$ of the larger square, while each of the small shaded central squares is $\frac{1}{9}$ of the middle square, itself $\frac{4}{16} = \frac{1}{4}$ of the larger square. The total fraction shaded is therefore $4 \times \frac{1}{16} + 3 \times \frac{1}{9} \times \frac{1}{4} = \frac{1}{4} + \frac{3}{36} = \frac{1}{4} + \frac{1}{12} = \frac{3}{12} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$. See the Notes below for an alternative and briefer 'in hindsight' explanation.

21 18 cm^2 Since the perimeter is 48 cm, the length of each side of the square is 12 cm. As can be seen in the diagram, the triangle forms $\frac{1}{8}$ of the area of the square, so its area will be $\frac{1}{8} \times 12 \times 12 = 18\,\text{cm}^2$.



22 108° The square has a right angle at Q, so we can deduce that angle $XQR = 180° - 21° - 90° = 69°$. We also know that the interior angle of the regular hexagon is 120°. So we can deduce that angle $XRQ = 180° - 120° - 21° = 39°$. Now, using the angle sum of triangle RXQ, angle $RXQ = 180° - 39° - 69° = 72°$. Therefore $x° = 180° - 72° = 108°$.



23 8 The percentage of spiders that are hairy-legged is strictly between 60% and 65%. If we think in fractions, the table below turns the simpler fractions into percentages:

	1	2	3	4	5	6
half	50%					
thirds	$33\frac{1}{3}$%	$66\frac{2}{3}$%				
quarters	25%	50%	75%			
fifths	20%	40%	60%	80%		
sixths	$16\frac{2}{3}$%	$33\frac{1}{3}$%	50%	$66\frac{2}{3}$%	$83\frac{1}{3}$%	
sevenths	≈ 14%	≈ 29%	≈ 43%	≈ 57%	≈ 71%	≈ 86%
eighths	12.5%	25%	37.5%	50%	**62.5%**	75%

The first fraction we discover that is strictly between 60% and 65% is $\frac{5}{8}$, and so the smallest possible number of spiders must be 8.

24 6 kg Each cat has been weighed twice, and so twice the combined weight of all three cats is $(5 + 8 + 9)$ kg = 22 kg; therefore their combined weight is 11 kg. The weight of the heaviest cat must be the difference between the total of the three and the least of the paired weights, that is 11 kg − 5 kg = 6 kg.

25 2245 Let Tabitha's four-digit number N be represented by the digits '$pqrs$'. If she rubbed out the digit p or the digit q or the digit r to get her new three-digit number M, both N and M would have the digit s in the units column and the result $N - M$ would end in $s - s$, a zero – whereas the units digit of 2021 is 1.

Therefore the digit Tabitha that rubbed out must have been the last one, s. This leads to the calculation shown on the right. One way of interpreting this is as $10 \times M + s - M = 9M + s = 2021$. Now the nearest multiple of 9 less than 2021 is 2016 and s is a single digit. This means that the only solution is $M = 2016 \div 9 = 224$ and $s = 5$. Hence Tabitha's number N is 2245.

$$\begin{array}{r} p\ q\ r\ s \\ -\ p\ q\ r \\ \hline 2\ 0\ 2\ 1 \end{array}$$

Some notes and possibilities for further problems

3 Humans display many similar ratios in the dimensions of various parts of their bodies. For most people, the length of one's arm-span is roughly equal to one's height. The *femur*, the bone between a person's hip and knee, is generally about a quarter of their height. For giraffes, the length of their neck is very nearly half of their entire height. Pupils could survey other correlations and ratios in relation to different body parts.

6 The longest days of this year, 2021, at Greenwich were between 17 June and 24 June, when the day, from sunrise to sunset, lasted 16 hours and 38 minutes. The shortest days will be between 18 December and 24 December, when the days will last only 7 hours and 50 minutes.

7 The world's oldest tortoise, Jonathan, is a Seychelles giant tortoise and is believed to have hatched around 1832, making him around 189 years old. He lives on the South Atlantic island of Saint Helena. Pupils may find it interesting to discover further examples of other long-living creatures or trees.

12 Another approach is to note that for the children, there are three ways to choose the child sitting left of centre; then once that child has been chosen, two ways to choose the child sitting in the centre, and then 1 choice (in effect, no choice) for the child to sit on the right of centre. So, altogether, there are $3 \times 2 \times 1 = 6$ ways in which the children can sit. As before, for each of these ways, there are 2 ways in which Pat and Sam can choose to sit, so giving a total of $2 \times 6 = 12$ ways.

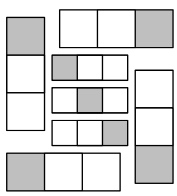
13 A variation on this type of question would be to find how far behind the slower walker is when the faster one finishes. Here, after 3 hours, Mr Waddle has walked 6 miles and so has 6 miles to go.

14 It is vital for pupils to recognise the significance in this question of the word *always*. It is certainly possible for the sum of three consecutive numbers to be a multiple of 5 ($4 + 5 + 6 = 15$), or odd ($2 + 3 + 4 = 9$), or prime ($0 + 1 + 2 = 3$), or even ($1 + 2 + 3 = 6$), but the total will *always* be a multiple of 3.

16 An alternative way to count the high-fives is to observe that each of the five girls makes four of them. Counting 5×4 would count each high-five twice, so the number is $\frac{1}{2} \times 5 \times 4 = 10$. The number of high-fives for any number of participants will hence be a triangular number.

17 Alternatively, using algebra, let it be y years ago that Isaac's age was one third of his mother's, or, equivalently, that her age was 3 times his. We can form the equation $81 - y = 3(53 - y)$. Hence $81 - y = 159 - 3y$, and so $2y = 159 - 81 = 78$, whence $y = 78 \div 2 = 39$. So it was 39 years ago, when Isaac was 14 and his mother was 42.

20 The alternative way of seeing that $\frac{1}{3}$ of the large square is shaded arises from observing that the square can be divided into rectangles, each of which is $\frac{1}{3}$ shaded. Taking this further, there is nothing special about either the fraction $\frac{1}{3}$ or the subdivisions into sets of congruent rectangles; one could divide the square in any way and, as long as $\frac{a}{b}$ of each constituent part was shaded, then $\frac{a}{b}$ of the entire square would be shaded.



25 It would be reasonable to ask whether it is possible to produce other four-digit numbers, or year numbers, by means of the same process: that is start with a four-digit number N, delete from it a digit to form a three-digit number, say M, and then subtract M from N. Certainly, adapting the method above for 2022, we have $2246 - 224 = 2022$. It would seem that the general method is to divide the required number, say Y, by 9, taking the whole number part, multiplying that by 10 and then adding the remainder obtained in the previous division. So wishing to obtain 5678, $5678 \div 9 = 630$ with a remainder of 8. Now $6308 - 630 = 5678$. To obtain an answer greater or equal to 9000, one would have to start with a five-digit number by this method, for example, $10\,836 - 1083 = 9753$. Alternatively, if the required final answer ends in 0, it might be possible to find other solutions: for example, $3564 - 354 = 3210$.